

# HEAT AND MASS TRANSFER IN THE MHD FLOW OF A VISCO-ELSTIC FLUID IN A ROTATING POROUS CHANNEL WITH RADIATIVE HEAT AND CHEMICAL REACTION

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## ABSTRACT

This paper deals with heat and mass transfer in the MHD of a visco-elastic fluid in a rotating porous channel with radiative heat and chemical reaction. The problem has been formulated with the physical conditions involved, subjected to the adequate boundary conditions. Walters' B' fluid model has been used to develop the equation of motion and the constitute equations of motion, energy and concentration have been solved with the help of complex function. Expressions for velocity, temperature and concentration are arrived at. Flow characteristics are known through graphs and tables drawn by numerical computation varying the values of fluid parameters. It is observed that the increase in radiation parameter decreases the secondary flow velocity and similar result is obtained in case of external magnetic field strength.

**Keywords :** Heat and Mass transfer, MHD, rotating porous channel, radiative heat, chemical reaction, non-Newtonian fluid.

## 1. INTRODUCTION

The Magnetohydrodynamic (MHD) free convection with heat transfer in a rotating system has been studied due to its importance in the design of magnetohydrodynamic (MHD) generators and accelerators in geophysics, in design of underground water energy storage system, soil sciences, astrophysics, nuclear power

reactors, MHD boundary layer control of re-entry vehicles and so on. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. On account of their varied importance, these flows have been studied by several authors. By taking constant suction and injection at the two porous plates of the channel, Attia and Kotb<sup>1</sup> analysed the MHD flow between two parallel porous plates. Nanda and Mohanty<sup>2</sup> studied the MHD flow between two infinite horizontal parallel plates in a rotating system.

The role of thermal radiation on the flow and heat transfer process is of importance in the design of many advanced energy conversion systems operating at higher temperatures. Thermal radiation within these systems is usually the result of emission of hot walls and the working fluid, Radiative convective flows are encountered in countless industrial and environment processes e.g., heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flow, solar power technology and space vehicle re-entry. In view of these important applications of radiative heat, recently Abdelkhalek<sup>3</sup> analysed the radiative and dissipation effect on unsteady MHD mixed convection laminar boundary layer flow of an electrically conducting viscous micropolar fluid past an infinite vertical plate. By taking variable temperature and uniform mass diffusion, Muthucumarswamy and Kulandaivel<sup>4</sup> studied the effect of thermal radiation on moving infinite vertical plate. However, the problem of fluid flows in rotating channel have received relatively less attention. Very recently Singh and Mathew<sup>5</sup> obtained an exact solution of a hydromagnetic oscillatory flow in a horizontal porous channel in a rotating system. Hussain and Mohammad<sup>6</sup> have studied the effect of Hall current on hydromagnetic free convection flow near an accelerated porous plate. Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux has been analysed by Acharya *et al*<sup>7</sup>. Effect of Hall current on hydromagnetic free convection flow near an exponentially accelerated porous plate with mass transfer has been studied by Dash and Rath. However, the problem on non-parallel vertex instability of natural convection flow over a non-isothermal inclined flat plate with simultaneous thermal and mass diffusion has been analysed by Singh *et al*<sup>9</sup>. The three dimensional free convection flow in a vertical channel filled with a porous medium has been obtained by Guria *et al*<sup>10</sup>. Dash *et al*<sup>11</sup> discussed the free convection MHD flow through porous

media of a rotating Oldroyd fluid past an infinite vertical porous plate with heat and mass transfer. Singh and Garg<sup>12</sup> have obtained the exact solution of an oscillatory free convective MHD flow in a rotating porous channel with radiative heat. Biswal and Pattnaik<sup>13</sup> have analysed MHD Couette flow in Oldroyd liquid.

Other works of studying the effect of Hall current on MHD flow include that of Biswal *et al.*<sup>14</sup> and Biswal and Sahoo<sup>15</sup>. However, MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction studied by Dash *et al.*<sup>16</sup>. Kumari and Nath<sup>17</sup> discussed the transient MHD mixed convection from a vertical surface moved impulsively from rest.

Our aim in this paper is to study heat and mass transfer in the MHD flow of a visco-elastic fluid in rotating porous channel with radiative heat and chemical reaction.

## 2. FORMULATION OF THE PROBLEM

Consider an oscillatory free convective flow of a visco-elastic incompressible fluid bounded between two infinite vertical porous plates distance  $d$  apart. A constant injection velocity  $W_0$  is applied at the stationary plate  $z^* = 0$  and the same constant suction velocity,  $W_0$  is applied at the plate  $z^* = d$ , which is oscillating in its own plane with a velocity  $U^*(t^*)$  about a non-zero constant mean velocity  $U_0$ . The origin is assumed to be at the plate  $z^* = 0$  and the channel is oriented vertically upward along the  $x^*$ -axis. The channel rotates as a rigid body with uniform angular velocity  $\Omega^*$  about the  $z^*$ -axis in the presence of a constant magnetic field  $B_0$  normal to the planes of the plates. The temperatures of the stationary and the moving plates are high enough to induce radiative heat transfer. It is also assumed that the radiation heat flux in the  $x^*$ -direction is negligible as compared to that in the  $z^*$ -direction. Since the plates are infinite in extent, all the physical quantities except the pressure, depend only on  $z^*$  and  $t^*$ . The equation of continuity

$$\nabla \cdot \vec{V} = 0 \quad (2.1)$$

gives on integration  $W^* = W_0$  where

$$\vec{V} = (u^*, v^*, w^*) \quad (2.2)$$

The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering

medium. Then by usual Boussinesq approximations, the flow of radiative fluid is governed by the following equations:

$$\frac{\partial u^*}{\partial t^*} + W_0 \frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + \frac{K_0}{\rho} + \frac{\partial^3 u^*}{\partial z^{*2} \partial t^*} + 2\Omega^* V^* + g\beta(T^* - T_d^*) + g\beta^*(C^* - C_d^*) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\nu u^*}{K^*} \quad (2.3)$$

$$\frac{\partial v^*}{\partial t^*} + W_0 \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} + \frac{K_0}{\rho} + \frac{\partial^3 v^*}{\partial z^{*2} \partial t^*} - 2\Omega^* U^* - \frac{\sigma B_0^2 v^*}{\rho} - \frac{\nu}{k^*} V^* \quad (2.4)$$

$$\frac{\partial T^*}{\partial t^*} + W_0 \frac{\partial T^*}{\partial z^*} = -\frac{k}{\rho C_p} \left[ \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{k} \frac{\partial q^*}{\partial z^*} \right] \quad (2.5)$$

$$\frac{\partial C^*}{\partial t^*} + W_0 \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} + \lambda^* \quad (2.6)$$

The boundary conditions of the problem are

$$\left. \begin{aligned} u^* = v^* = 0, T^* = T_0 + \varepsilon (T_0^* - T_d^*) \cos \omega^* t^*, \\ C^* = C_0 + \varepsilon (C_0^* - C_d^*) \cos \omega^* t^*, \text{ at } z^* = 0, \\ u^* = U^*(t^*) = U_0 (1 + \varepsilon \cos \omega^* t^*), V^* = 0, \\ T^* = T_d^*, C^* = C_d^*, \text{ at } z^* = d. \end{aligned} \right\} \quad (2.7)$$

Where  $\nu$  is the kinematic viscosity,  $t^*$  is the time,  $\rho$  is the density and  $P^*$  is the modified pressure,  $B_0 (= \mu_e H_0)$  the electromagnetic induction,  $\mu_e$  the magnetic permeability,  $H_0$  the intensity of magnetic field,  $\sigma$  the conductivity of the fluid,  $T^*$  is the temperature,  $C_p$  is the specific heat at constant pressure,  $k$  the thermal conductivity,  $g$  is the acceleration due to gravity,  $b$  the coefficient of volume expansion and  $q$  is the radiative heat.

Since the medium is optically thin with relatively low density, the radiative heat flux for the case becomes

$$\frac{\partial q^*}{\partial z^*} = -4\alpha\sigma^*(T_d^{*4} - T^{*4}) \quad (2.8a)$$

Where  $\alpha$  is the mean radiative absorption coefficient and  $\sigma^*$  is the Stefan-Boltzmann constant.

We assume that the temperature differences with the flow are sufficiently small such that  $T^{*4}$  may be expressed as a linear function of the temperature. This is accomplished by expanding  $T^{*4}$  in a Taylor series about  $Td^*$  and neglecting higher order terms, thus

$$i) \quad T^{*4} = 4Td^{*3} T^* - 3Td^{*4} \quad (2.8b)$$

By using equation (2.8a) and (2.8b), equation (2.5) reduces to

$$ii) \quad \frac{\partial T^*}{\partial t^*} + W_0 \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{16\alpha Td^{*3} \sigma^*}{\rho C_p} (T^* - Td^*) \quad (2.9)$$

The reaction term in equn. (2.6) takes the form

$$\lambda^* = -k^* (C - C_d^*)$$

Introducing the following non-dimensional quantities

$$\eta = \frac{z^*}{d}, \quad t = \omega^* t^*, \quad u = \frac{u^*}{U_0}, \quad V = \frac{V^*}{U_0},$$

$$\omega = \frac{\omega^* d^2}{\nu}, \quad \Omega = \Omega^* \frac{d^2}{\nu}, \quad U = \frac{U^*}{U_0},$$

$$\lambda = \frac{W_0 d}{\nu}, \quad \theta = \frac{T^* - Td^*}{T_0^* - Td^*},$$

$$C = \frac{C^* - C_d^*}{C_0^* - C_d^*}, \quad G_r = \frac{\nu g \beta (T_0^* - Td^*)}{U_0^2 W_0},$$

$$G_c = \frac{\nu g \beta^* (C_0^* - C_d^*)}{U_0^2 W_0}, \quad P_r = \frac{\mu C_p}{k},$$

$$R_c = \frac{k_0}{\rho d^2}, \quad R = \frac{16\alpha \sigma^* d^2 Td^{*3}}{T_0^* - Td^*}, \quad M = B_0 d \sqrt{\frac{\sigma}{\mu}},$$

$$S_c = \frac{\nu}{D}, \quad k_p = \frac{U_0^2 k^*}{\nu^2}, \quad k = \frac{VK^*}{U_0^2},$$

Where  $p^*$  is the modified pressure that includes centrifugal force,  $\nu$  is the kinematic viscosity,  $\omega$  is the frequency parameter,  $\Omega$  is the rotation parameter,  $\lambda$  is the suction parameter,  $G_r$  is the thermal Grashof number,  $P_r$  is the Prandtl number,  $R$  is the radiative heat flux and  $M$  is the Hartmann number,  $\mu$  is the coefficient of viscosity,  $R_c$

is the non-Newtonian parameter,  $G_c$  is the mass diffusion Grashof number,  $S_c$  is the Schmidt number,  $k_p$  is the permeability parameter,  $\theta$  is the non-dimensional temperature,  $C$  is the non-dimensional concentration, into equation (3), (4), (6) and (9), we get.

$$\omega = \frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial \eta} = \omega \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial \eta^2} + 2\Omega v + \lambda^2 G_r \theta - M^2(u - U) + \omega R_c \frac{\partial^3 u}{\partial \eta^2 \partial t} + \lambda^2 G_c C - \frac{1}{k_p} u \quad (2.11)$$

$$\omega = \frac{\partial v}{\partial t} + \lambda \frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} - 2\Omega(u - U) - \left( M^2 + \frac{1}{K_p} \right) v + \omega R_c \frac{\partial^2 v}{\partial \eta^2 \partial t} \quad (2.12)$$

$$\omega = \frac{\partial \theta}{\partial t} + \lambda \frac{\partial \theta}{\partial \eta} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial \eta^2} - R\theta \quad (2.13)$$

$$\omega = \frac{\partial C}{\partial t} + \lambda \frac{\partial C}{\partial \eta} + Kc = \frac{1}{S_c} \frac{\partial^2 C}{\partial \eta^2} \quad (2.14)$$

Introducing the complex velocity  $F = u + iv$  we find that equations (2.11) and (2.12) can be combined into a single equation of the form

$$\omega = \frac{\partial F}{\partial t} + \lambda \frac{\partial F}{\partial \eta} = \omega \frac{\partial U}{\partial t} + \frac{\partial^2 F}{\partial \eta^2} + \lambda^2 G_r \theta + \lambda^2 G_c C - 2i\Omega (F - U) - \frac{1}{K_p} F \quad (2.15)$$

The transformed boundary conditions become

$$\left. \begin{aligned} F = 0, \theta = 1 + \frac{\epsilon}{2}(e^{it} + e^{-it}), \\ C = 1 + \frac{\epsilon}{2}(e^{it} + e^{-it}) \text{ at } \eta = 0 \\ F = U(t) = 1 + \frac{\epsilon}{2}(e^{it} + e^{-it}), \\ C = 0, \theta = 0, \text{ at } \eta = 1 \end{aligned} \right\} \quad (2.16)$$

### 3. SOLUTIONS OF THE EQUATIONS

In order to solve the equation (2.15), (2.13) and (2.14), we assume the solution of the form

$$F(\eta, t) = F_0(\eta) + \frac{\epsilon}{2} \{F_1(\eta) e^{it} + F_2(\eta) e^{-it}\} \quad (3.1)$$

$$\theta(\eta, t) = \theta_0(\eta) + \frac{\epsilon}{2} \{\theta_1(\eta) e^{it} + \theta_2(\eta) e^{-it}\} \quad (3.2)$$

$$C(\eta, t) = C_0(\eta) + \frac{\epsilon}{2} \{C_1(\eta) e^{it} + C_2(\eta) e^{-it}\} \quad (3.3)$$

Substituting (3.1), (3.2) and (3.3) in the equations (2.15), (2.13) and (4.2.14) respectively and then comparing the harmonic and non-harmonic terms, we obtain

$$F_0'' - \eta_1 F_0' - \eta_2 F_0 = -\eta_2 - \eta_3 \theta_0 - \eta_4 C_0 \quad (3.4)$$

$$F_1'' - \eta_1 F_1' - \eta_5 F_1 = -\eta_5 - \eta_3 \theta_1 - \eta_4 C_1 \quad (3.5)$$

$$F_2'' - \eta_1 F_2' - \eta_6 F_2 = -\eta_6 - \eta_3 \theta_2 - \eta_4 C_2 \quad (3.6)$$

$$\theta_0'' - \eta_7 \theta_0' - \eta_8 \theta_0 = 0 \quad (3.7)$$

$$\theta_1'' - \eta_7 \theta_1' - \eta_9 \theta_1 = 0 \quad (3.8)$$

$$\theta_2'' - \eta_7 \theta_2' - \eta_{10} \theta_2 = 0 \quad (3.9)$$

$$C_0'' - \eta_{11} C_0' = 0 \quad (3.10)$$

$$C_1'' - \eta_1 C_1' - \eta_{12} C_1 = 0 \quad (3.11)$$

$$C_2'' - \eta_{11} C_2' + \eta_{12} C_2 = 0 \quad (3.12)$$

The corresponding transformed boundary conditions are

$$\begin{aligned} F_0 = F_1 = F_2 = 0, \theta_0 = \theta_1 = \theta_2 = 1, \\ C_0 = C_1 = C_2 = 1, \text{ at } \eta = 0 \\ F_0 = F_1 = F_2 = 1, \theta_0 = \theta_1 = \theta_2 = 0, \\ C_0 = C_1 = C_2 = 2, \text{ at } \eta = 1 \end{aligned} \quad (3.13)$$

Solving equations (3.4) – (3.12) under the boundary condition (3.13), we get

$$F_0 = A_6 e^{m_7 \eta} + A_7 e^{m_8 \eta} + A_{13} - A_{14} e^{m_1 \eta} - A_{15} e^{m_2 \eta} - A_{16} e^{m_{11} \eta} \quad (3.14)$$

$$\begin{aligned} F_1 = A_{22} e^{m_{13} \eta} + A_{23} e^{m_{14} \eta} + A_{29} e^{m_3 \eta} \\ + A_{30} e^{m_4 \eta} - A_{31} e^{m_{17} \eta} - A_{32} e^{m_{12} \eta} \end{aligned} \quad (3.15)$$

$$F_2 = A_{38} e^{m_{15}\eta} + A_{39} e^{m_{16}\eta} + A_{45} e^{m_{15}\eta} + A_{46} e^{m_6\eta} - A_{47} e^{m_{13}\eta} - A_{48} e^{m_{14}\eta} \quad (3.16)$$

$$\theta_0 = z_1 e^{m_1\eta} + z_2 e^{m_2\eta} \quad (3.17)$$

$$\theta_1 = z_3 e^{m_3\eta} + z_4 e^{m_4\eta} \quad (3.18)$$

$$\theta_2 = z_5 e^{m_5\eta} + z_6 e^{m_6\eta} \quad (3.19)$$

$$C_0 = A_0 + A_1 e^{\eta_1\eta} \quad (3.20)$$

$$C_1 = A_2 e^{m_{11}\eta} + A_3 e^{m_{12}\eta} \quad (3.21)$$

$$C_2 = A_4 e^{m_{13}\eta} + A_5 e^{m_{14}\eta} \quad (3.22)$$

**Skin-frictions:**

Skin-friction for the steady part of the flow of the lower plate of the channel is given by

$$\tau_0 = \left. \frac{\partial u_0}{\partial \eta} \right|_{\eta=0} + R_c \left. \frac{\partial^2 u_0}{\partial \eta^2} \right|_{\eta=0} \quad (3.23)$$

$$\tau_0 = A_6 m_7 + A_7 mg - A_{14} m_1 - A_{15} m_2 - A_{16} \eta_{11} + R_c \left( A_6 m_7^2 + A_7 m_8^2 - A_{14} m_1^2 - A_{15} m_2^2 - A_{16} \eta_{11}^2 \right) \quad (3.24)$$

Skin-friction at the upper plate of the channel for the velocity component  $v_0$  is given by

$$\tau_1 = \left. \frac{\partial v_0}{\partial \eta} \right|_{\eta=l} + R_c \left. \frac{\partial^2 v_0}{\partial \eta^2} \right|_{\eta=l} \quad (3.25)$$

$$\tau_1 = A_6 m_7 e^{m_7} + A_7 mg e^{mg} - A_{14} m_1 e^{m_1} - A_{15} m_2 e^{m_2} - A_{16} \eta_{11} e^{\eta_{11}} + R_c \left( A_6 m_7^2 e^{m_7} + A_7 m_8^2 e^{m_8} - A_{14} m_1^2 e^{m_1} - A_{15} m_2^2 e^{m_2} - A_{16} \eta_{11}^2 e^{\eta_{11}} \right) \quad (3.26)$$

**Rates of heat transfer:**

For the steady part of the flow, the rate of heat transfer at the lower plate of the channel is given by

$$Nu_0 = - \left. \frac{\partial \theta_0}{\partial \eta} \right|_{\eta=0} = - (z_1 m_1 + z_2 m_2) \quad (3.27)$$



For the steady part of the flow, the rate of heat transfer at the upper plate of the channel is given by

$$Nu_1 = -\left. \frac{\partial \theta_0}{\partial \eta} \right|_{\eta=1} \quad (3.28)$$

$$= - (z_1 m_1 e^{m_1} + z_2 m_2 e^{m_2})$$

**Concentration gradients:**

For the steady part of the flow, the concentration gradient at the lower plate is

$$CG_0 = -\left. \frac{\partial C_0}{\partial \eta} \right|_{\eta=0} = - (A_1 \eta_{11}) \quad (3.29)$$

For the steady part of the flow, the concentration gradient at the upper plate is

$$CG_1 = -\left. \frac{\partial C_0}{\partial \eta} \right|_{\eta=1} = - (A_1 \eta_{11} e^{\eta_{11}}) \quad (3.30)$$

The constants involved in the above equations are not given here in order to save space.

**4. RESULTS AND DISCUSSION**

Heat and mass transfer in the MHD flow of visco-elastic fluid in a rotating porous channel with radiative heat have been studied through graphs and tables. The effects of various fluid parameters like non-Newtonian parameter ( $R_c$ ), rotation parameter ( $\Omega$ ), Hartmann number ( $M$ ), Grashof number ( $G_r$ ), modified Grashof number ( $G_c$ ), suction parameter ( $\lambda$ ), permeability parameter ( $K_p$ ), Prandtl number ( $P_r$ ), radiative heat flux ( $R$ ), frequency parameter ( $\omega$ ) and the Schmidt number ( $S_c$ ) on the non-Newtonian flow under investigation have been found. The steady part of the flow is presented by  $f_0$  and the unsteady part of the flow has two components  $f_1$  and  $f_2$ . Here, we have discussed the each part of the flow separately.

**Velocity of flow:**

Figure 1 explains the effects of  $G_r$ ,  $G_c$ ,  $R$ ,  $\lambda$  and  $M$  on the steady part of the flow ( $f_0$ ) keeping the other parameters fixed. Taking a particular visco-elastic fluid for which  $R_c=0.01$ , the profiles have been plotted. It is observed that the steady component of the flow velocity rises with the Grashof number  $G_r$  and the same effect is marked for  $G_c$ , But the radiation parameter reduces the flow velocity reversing the profile shape (curve V). Suction parameter  $\lambda$  with the high transverse magnetic field

strength further decreases the flow velocity attending negative values below the channel length  $\eta=0.5$ .

Figure 2 exhibits the effects of various fluid parameters on the secondary velocity component  $f_1$  keeping  $P_r$ ,  $t$ ,  $S_c$  and  $\omega$  fixed. It is observed that the Grashof number  $G_r$  increases the  $f_1$  (curves I, II, III) while the modified Grashof number  $G_c$  decreases the velocity component  $f_1$ . This is quite natural that the rise in the value of  $G_r$  means the rise in the value of fluid temperature that causes the fluid molecules more energetic. So, the speeds of the molecules becomes more causing the fluid velocity  $f_1$  to rise. But increase in the value of  $G_c$  means the rise in the species concentration. More mass diffusion absorbs more heat energy and hence the fluid molecules becomes cooled causing their speed slower. As a result, the unsteady part of the fluid velocity  $f_1$  is reduced. Again, the rise in the radiation parameter  $R$  reduces the fluid velocity  $f_1$  further. This is due to the fact that if more heat energy is radiated, the fluid molecules becomes less energetic and their motion becomes slower causing  $f_1$  to decrease. Raising the external magnetic field strength, the fluid velocity  $f_1$  rises for the rotation parameter  $\Omega=50$ . The fluid velocity  $f_1$  decreases with the rise in the value of non-Newtonian parameter  $R_c$  (curves VII and VIII).

Effects of non-Newtonian parameter  $R_c$ , thermal Grashof number  $G_r$ , modified Grashof number  $G_c$ , radiation parameter  $R$ , suction parameter  $\lambda$ , magnetic parameter  $M$ , rotation parameter  $\Omega$  and the frequency parameter  $\omega$  on the secondary velocity part  $f_2$  of the micro-elastic flow have been presented by the profiles of Fig.3. It is observed that the increase in  $G_r$  decelerates the flow and rise in  $G_c$  further reduces the flow velocity. However, the increase in radiation parameter accelerates the flow. The rise in the external magnetic field strength reduces the flow velocity  $f_2$  (curve VI), beyond the channel length  $\eta$  slightly greater than 0.2, but below that the flow velocity  $f_2$  is increased. Increase in rotation parameter increases the flow velocity  $f_2$  slowly. Similar effect is marked in case of frequency parameter  $\omega$  and the non-Newtonian parameter  $R_c$ .

Figure 4 shows the effects of suction parameter  $\lambda$  and Prandtl number  $P_r$  on the steady temperature  $\theta_0$ . It is observed that  $\theta_0$  falls with the rise of both  $\lambda$  and  $P_r$ .

The effects of suction parameter  $\lambda$  and Schmidt number  $S_c$  on the mean concentration  $C_0$  have been illustrated in the Fig.5. It is observed that  $C_0$  decreases with the increase of  $\lambda$  as well as  $S_c$ , attaining always negative values. Generally  $C_0$

risers as the channel length  $\eta$  increases.

**Skin-friction:**

The values of the skin-friction  $\tau_0$  and  $\tau_1$  of the steady part of the flow are entered in the Table 1 for various values of the fluid parameter  $G_r$ ,  $G_c$ ,  $\lambda$ ,  $M$ ,  $R$  and  $\Omega$ . The increase in Grashof number ( $G_r$ ) increases the skin-friction  $\tau_0$  for both Newtonian ( $R_c=0.0$ ) and non-Newtonian ( $R_c \neq 0$ ) flow, but the opposite effect is marked in case of  $\tau_1$ . Increase in Grashof number decreases  $\tau_0$  and  $\tau_1$  for both Newtonian and non-Newtonian flow. Increase in suction parameter  $\lambda$  decreases  $\tau_0$  rapidly but increases  $\tau_1$  swiftly. Increase in the Hartmann number  $M$  increases  $\tau_0$  but decreases  $\tau_1$ .

**Table 1**

Values of the skin-frictions  $\tau_0$  &  $\tau_1$  of the steady part of the flow for  $R = 1, \Omega=25$

$G_r$	$G_c$	$\lambda$	$M$	$\tau_0$			$\tau_1$		
				$R_c$			$R_c$		
				0.00	0.01	0.05	0.00	0.01	0.05
5	2	2	2	1.4605	1.4099	1.2073	2.1047	2.2069	2.6154
10	2	2	2	2.7646	2.6605	2.2442	1.7068	1.8373	2.3594
15	2	2	2	4.0687	3.9112	3.2810	1.3088	1.4678	2.1034
15	4	2	2	4.0139	3.8550	3.2192	-0.8857	-0.8143	-0.5289
15	4	3	2	-317.6913	-303.2010	-245.2397	414.3880	432.6485	505.6903
15	4	3	4	-0.7183	-0.6854	-0.5540	7.5892	8.3089	11.1879

**Rate of heat transfer:**

Table 2 shows the effects of  $P_r$ ,  $\lambda$ ,  $R$  and  $\omega$  on the mean rate of heat transfer and the rate of heat transfer for the steady parts of the flow. It is observed that the increase in the suction parameter ( $\lambda$ ) increases  $Nu_0$  but reduces  $Nu_1$ . Similar effect is marked with the rise of Prandtl number ( $P_r$ ). Increase in radiation parameter does show any effect on the mean rate of heat transfer as well as on the rate of heat transfer and same is the case with the frequency parameter ( $\omega$ ).

**Table 2**

Value of the rate of heat transfer  $Nu_0$  &  $Nu_1$  for the steady part of the flow

$\lambda$	$P_r$	$R$	$\omega$	$Nu_0$	$Nu_1$
2	9	1	5	2.42495	0.87195
3	9	1	5	3.26639	0.43995
3	12	1	5	4.00003	0.23810
3	12	2	5	4.00003	0.23810
3	12	2	10	4.00003	0.23810

**Concentration gradient:**

Table. 3 contains the values of mean concentration gradient and the concentration gradient for the steady part of the flow pertaining to various values of  $\lambda$ ,  $\omega$  and  $S_c$ . It is observed that the rise in the value of suction parameter ( $\lambda$ ) reduces  $CG_0$  but increases  $CG_1$ . However, the frequency parameter ( $\omega$ ) does not produce any effect on both these concentration gradient. The increase in the value of Schmidt number ( $S_c$ ) decreases the mean concentration gradient ( $CG_0$ ) very rapidly and which attains zero value for  $S_c=6.0$ , but  $CG_1$  rises with  $S_c$ .

**Table 3**

Value of the concentration gradient  $CG_0$  and  $CG_1$  for the steady part of the flow

$\lambda$	$\omega$	$S_c$	$CG_0$	$CG_1$
2	5	2	0.07194	3.92806
3	5	2	0.01484	5.98516
3	10	2	0.01484	5.98516
3	10	4	0.00007	11.99993
3	10	6	0.00000	18.00000

## CONCLUSION:

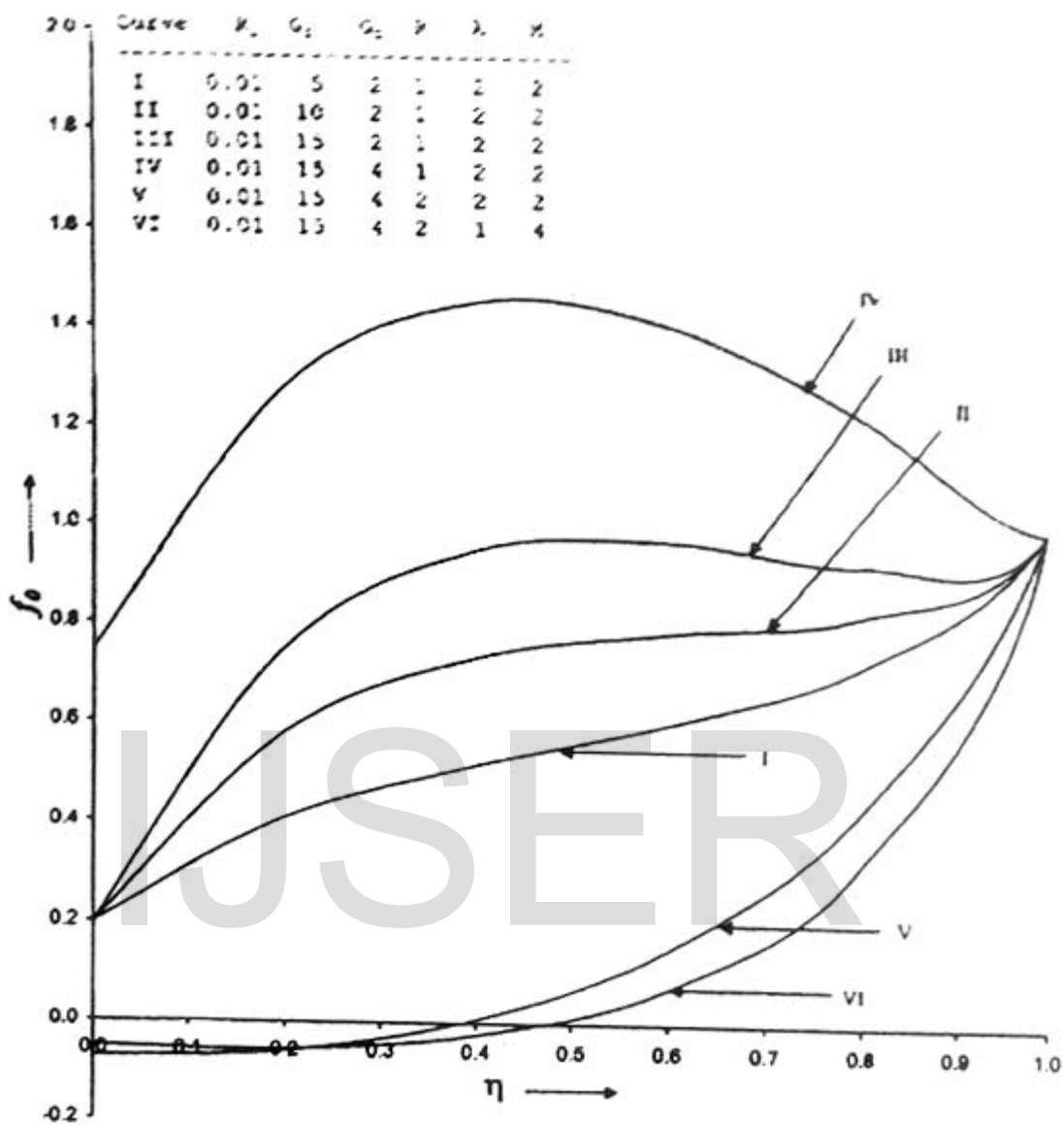
Above investigations on the heat and mass transfer in the MHD flow of a Walters'  $B'$  fluid in a rotating porous channel reveal the following significant findings.

- i) The steady component of the flow velocity rises with the thermal Grashof number  $G_r$  and mass concentration Grashof number ( $G_c$ )
- ii) The steady part of the flow velocity reduces with the suction parameter ( $\lambda$ ), radiation parameter ( $R$ ) and the magnetic parameter ( $M$ ).
- iii) The secondary flow velocity ( $f_1$ ) increases with  $G_r$  but decreases with  $G_c$ .
- iv) Increase in radiation parameter decreases the secondary flow velocity ( $f_1$ ).
- v) The secondary flow velocity ( $f_1$ ) reduces with the rise in the value of non-Newtonian parameter ( $R_c$ ).
- vi) The rise in the external magnetic field strength reduces the secondary flow velocity component  $f_2$ .
- vii) Steady temperature ( $\theta_0$ ) falls with the rise of both the suction parameter  $l$  and Prandtl number ( $P_r$ ).
- viii) Mean concentration ( $C_0$ ) decreases with the increase of both the suction parameter ( $\lambda$ ) and the Schmidt number ( $S_c$ ).

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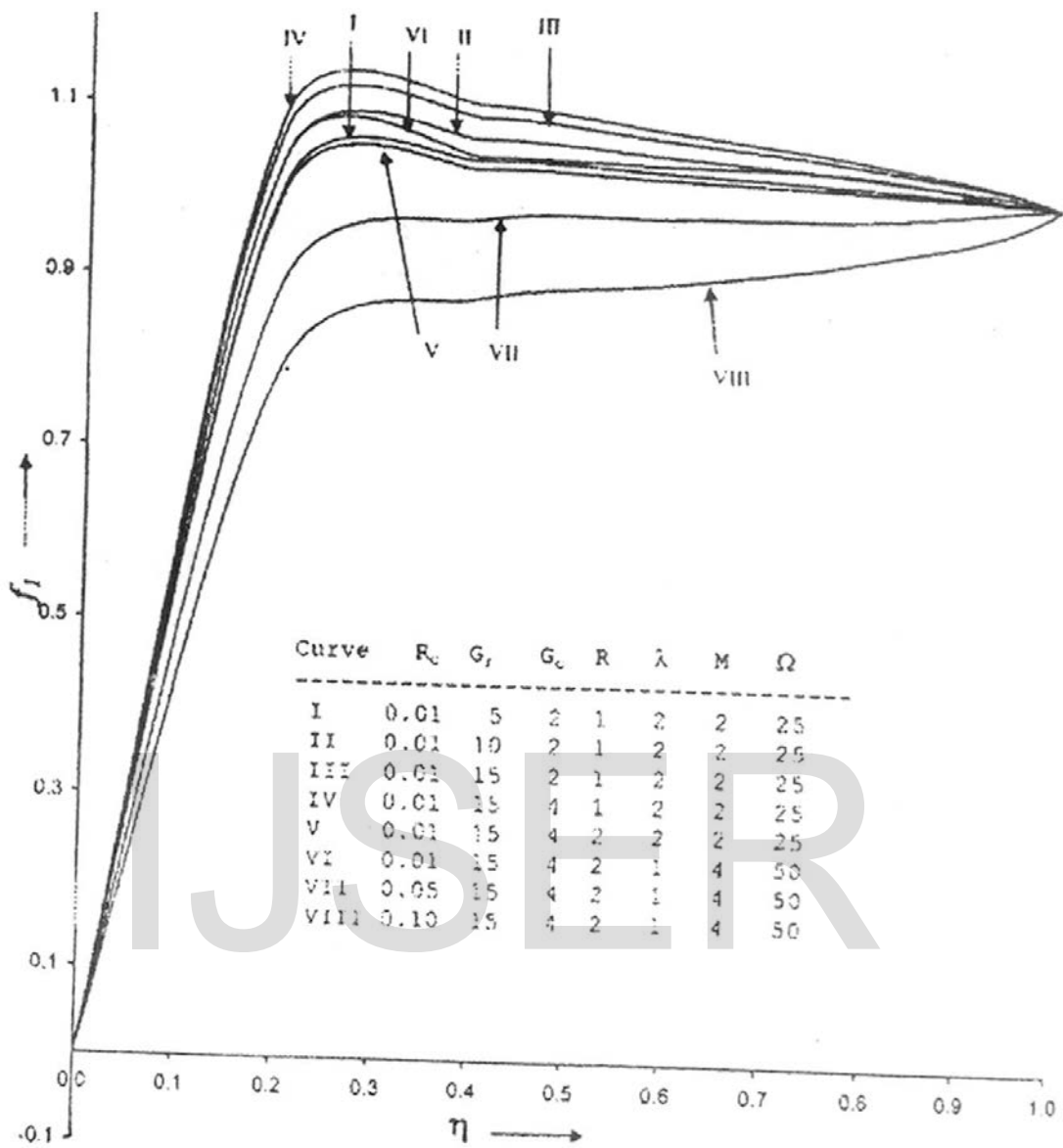
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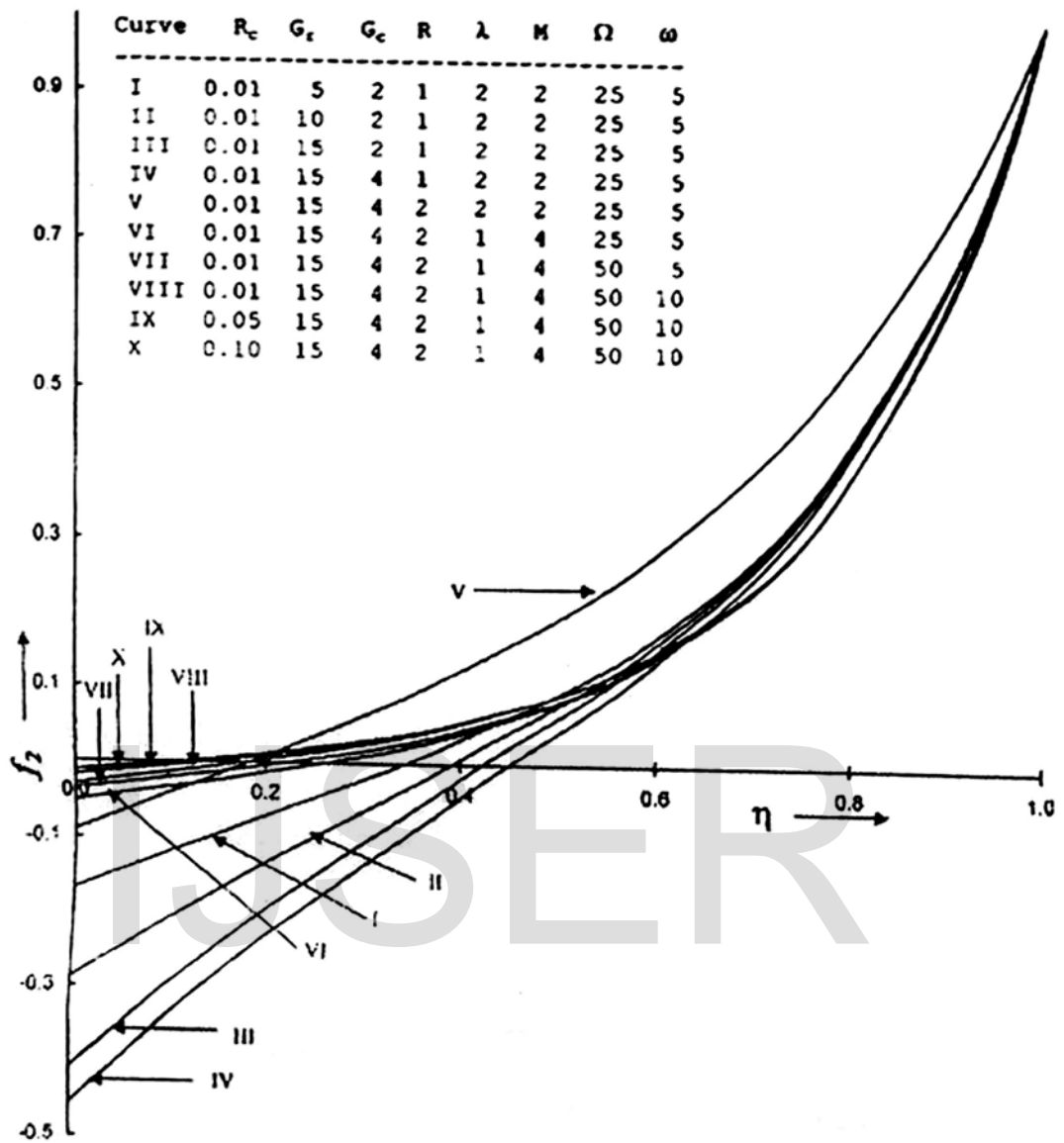


**Fig. 1 :** Effect of different parameters on  $f_0$  profile for  $P_r=9.0$ ,  $t=30$ ,  $S_c=2.0$ ,  $\Omega=2.5$ ,  $\omega=5$

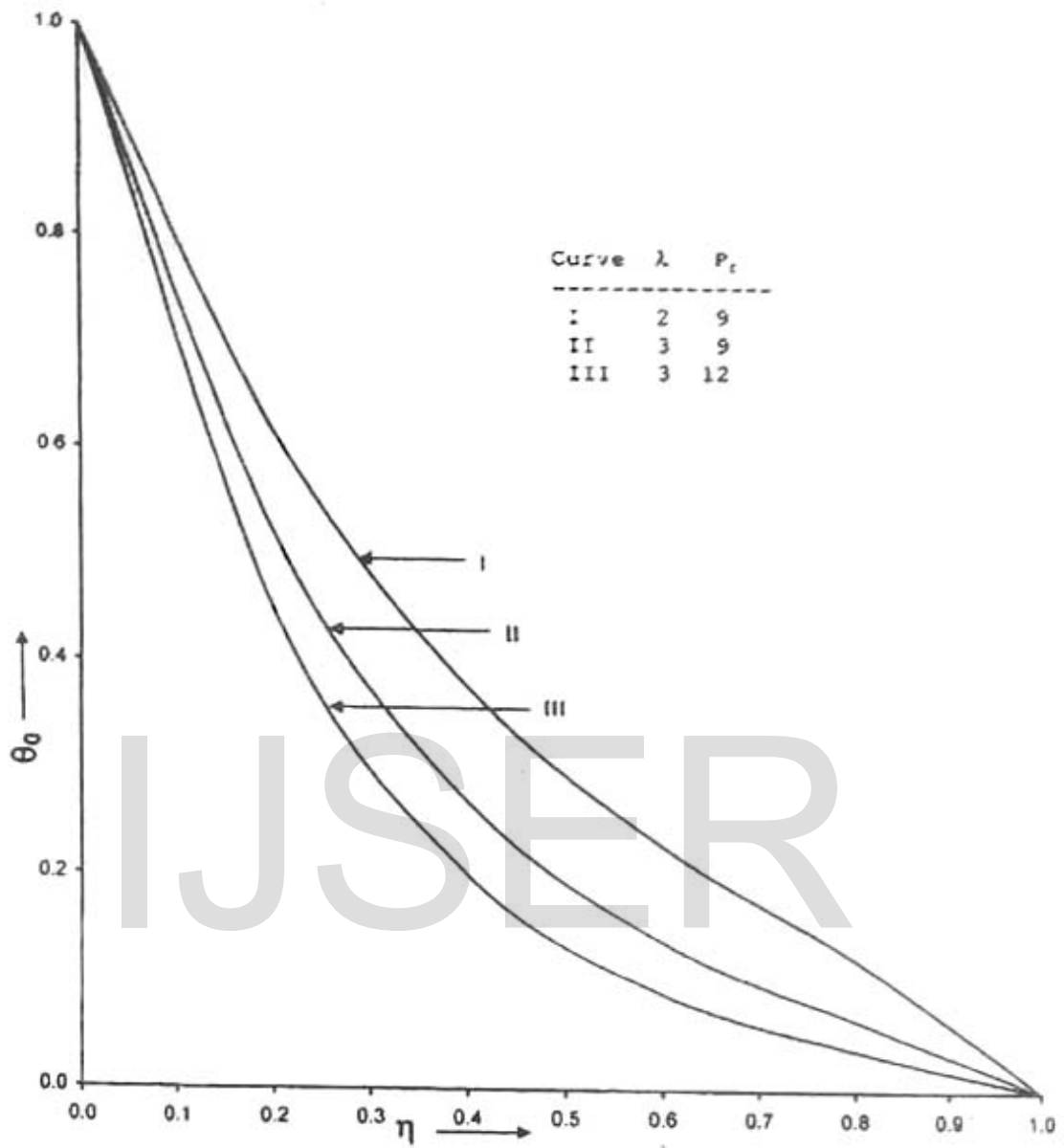




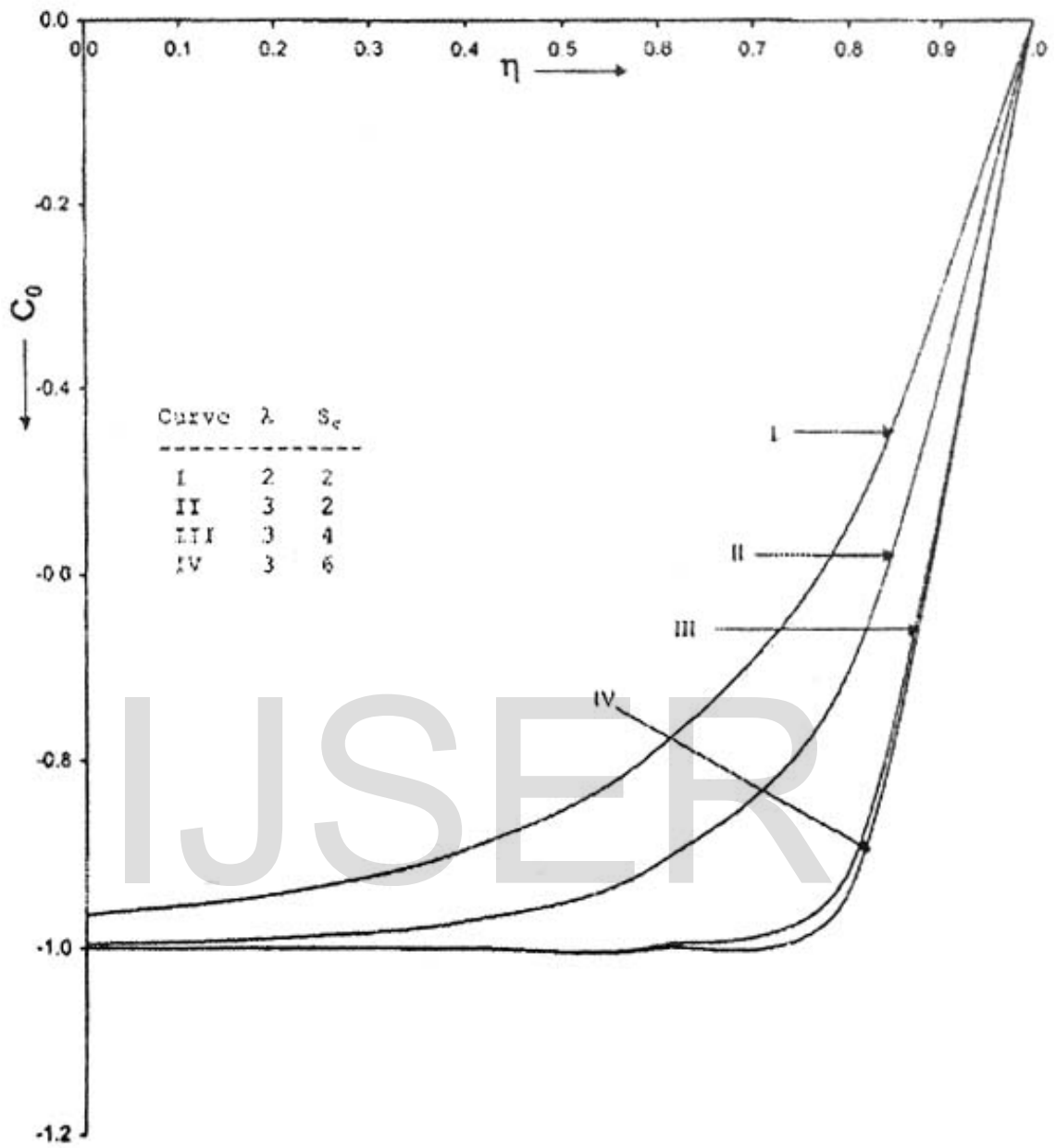
**Fig. 2 :** Effect of different parameters on  $f_1$  profile for  $P_r=9.0$ ,  $t=30$ ,  $S_c=2.0$ ,  $\Omega=2.5$ ,  $\omega=5$



**Fig. 3 :** Effect of different parameters on  $f_2$  profile for  $P_r=9.0$ ,  $t=30$ ,  $S_c=2.0$ .



**Fig. 4 :** Temperature profile ( $\theta_0$ ) for  $R_c = 0.01$ ,  $G_r=5$ ,  $G_c=2$ ,  $M=2$ ,  $t=1$ ,  $S_c=2.0$ ,  $\Omega=2.5$ ,  $R=1$ ,  $\omega=5$



**Fig. 5 :** Concentration profile ( $C_0$ ) for  $P_r = 9$ ,  $G_r=5$ ,  $G_c=2$ ,  $R=1$ ,  $t=1$ ,  $\Omega=2.5$ ,  $R=1$ ,  $\omega=5$